# Chapter 9: Transformational Geometry

SECTION 4: COMPOSITIONS OF TRANSFORMATIONS

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## I Can

- Apply theorems about isometries
- Identify and draw compositions of transformations (glide reflections)

## Isometry

THREE TYPES OF TRANSFORMATIONS:

1. Reflections (flipping across a line)
2. Translations (sliding along a vector)
3. Rotations (turning around a point)

A **composition of transformations** is one transformation followed by another.

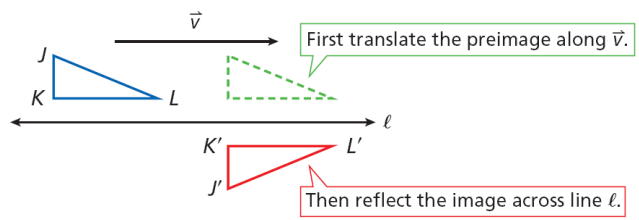
**Glide reflections** are the composition of a translation and a reflection

**Theorem 12-4-1**

A composition of two isometries is an isometry.

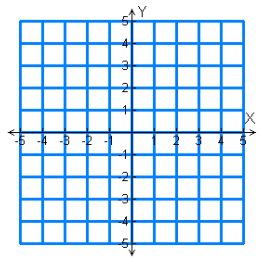
## Glide Reflections

The glide reflection that maps $\Delta JKL$ to $\Delta J'K'L'$ is the composition of a translation along $\vec{v}$ followed by a reflection across line $\ell$.



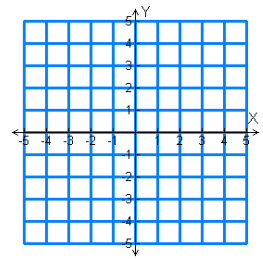
## Example

**Δ*KLM* has vertices *K*(4, −1), *L*(5, −2), and *M*(1, −4). Rotate Δ*KLM* 180° about the origin and then reflect it across the *y*-axis.**




## Example

**Δ*JKL* has vertices *J*(1,−2), *K*(4, −2), and *L*(3, 0). Reflect Δ*JKL* across the *x*-axis and then rotate it 180° about the origin.**

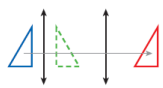



## Theorems

**Theorem 12-4-2**

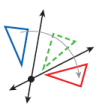
The composition of two reflections across two parallel lines is equivalent to a translation.

- The translation vector is perpendicular to the lines.
- The length of the translation vector is twice the distance between the lines.



The composition of two reflections across two intersecting lines is equivalent to a rotation.

- The center of rotation is the intersection of the lines.
- The angle of rotation is twice the measure of the angle formed by the lines.

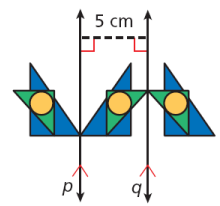


**Theorem 12-4-3**

Any translation or rotation is equivalent to a composition of two reflections.

## Example

**Sean reflects a design across line *p* and then reflects the image across line *q*. Describe a single transformation that moves the design from the original position to the final position.**



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